Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 18 November 2019 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]

2.　[Maximum mark: 6]

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events "fluent in Spanish" and "fluent in French".

The values m, n, p and q represent numbers of students.
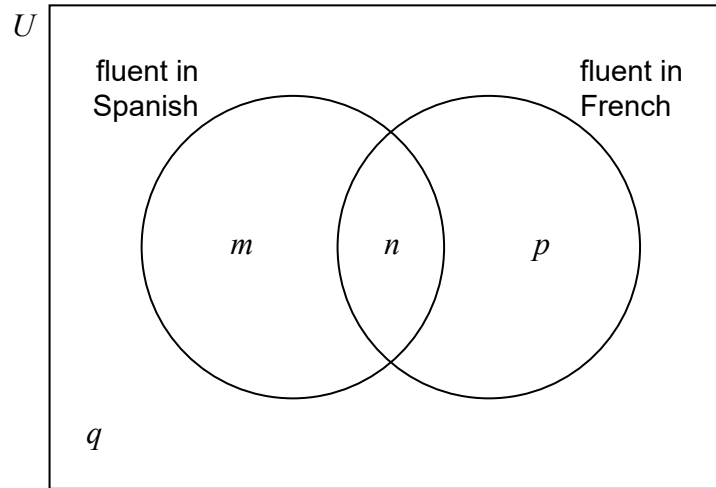
(a)　Write down the value of q.　　　　　　　　　　　　　　　　　　　　[1]

(b)　Find the value of n.　　　　　　　　　　　　　　　　　　　　　　　[2]

(c)　Write down the value of m and of p.　　　　　　　　　　　　　　　[3]

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3.   [Maximum mark: 7]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

(a)   Find the value of b.   [3]

(b)   The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation.   [4]

4.   [Maximum mark: 6]

Consider $\begin{pmatrix} 11 \\ a \end{pmatrix} = \dfrac{11!}{a!\,9!}$.

(a)   Find the value of a . [2]

(b)   Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x + 3)^{11}$. [4]

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Turn over

5.   [Maximum mark: 6]

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where x, $k \in \mathbb{R}$.

(a)   Show that the discriminant of $f'(x)$ is $k^2 - 16$.   [2]

(b)   Given that f is an increasing function, find all possible values of k.   [4]

6.   [Maximum mark: 8]

Let $f(x) = 4\cos\left(\dfrac{x}{2}\right) + 1$, for $0 \le x \le 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$.

7. [Maximum mark: 6]

Let X and Y be normally distributed with $X \sim \mathrm{N}(14, a^2)$ and $Y \sim \mathrm{N}(22, a^2)$, $a > 0$.

(a) Find b so that $\mathrm{P}(X > b) = \mathrm{P}(Y < b)$. [2]

It is given that $\mathrm{P}(X > 20) = 0.112$.

(b) Find $\mathrm{P}(16 < Y < 28)$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8.   [Maximum mark: 14]

A small cuboid box has a rectangular base of length $3x$ cm and width x cm, where $x > 0$.
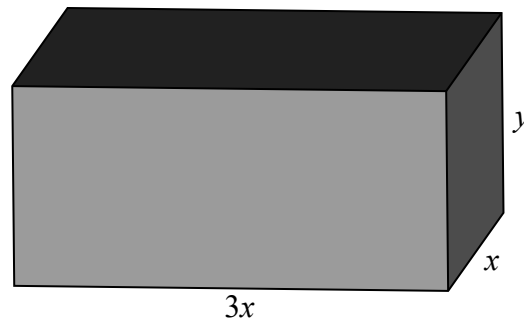The height is y cm, where $y > 0$.

diagram not to scale

The sum of the length, width and height is 12 cm.

(a)   Write down an expression for y in terms of x.   [1]

The volume of the box is V cm^3.

(b)   Find an expression for V in terms of x.   [2]

(c)   Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$.   [2]

(d)   (i)   Find the value of x for which V is a maximum.

        (ii)   Justify your answer.   [7]

(e)   Find the maximum volume.   [2]

Do **not** write solutions on this page.

9.   [Maximum mark: 17]

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

(a)   Find, in terms of k,

   (i) $\overrightarrow{OA} \cdot \overrightarrow{OC}$;

   (ii) $\overrightarrow{OB} \cdot \overrightarrow{OC}$.   [3]

(b)   Given that $A\hat{O}C = B\hat{O}C$, show that $k = 7$.   [8]

(c)   Calculate the area of triangle AOC.   [6]

Do **not** write solutions on this page.

10.   [Maximum mark: 14]

Let $g(x) = p^x + q$, for x, p, $q \in \mathbb{R}$, $p > 1$. The point $A(0, a)$ lies on the graph of g.

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

(a)   Write down the coordinates of B.   [2]

The line L_1 is tangent to the graph of f at B.

(b)   Given that $f'(a) = \dfrac{1}{\ln p}$, find the equation of L_1 **in terms of** x, p and q.   [5]

The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

The line L_2 passes through the point $(-2, -2)$.

The gradient of the normal to g at A is $\dfrac{1}{\ln\left(\dfrac{1}{3}\right)}$.

(c)   Find the equation of L_1 in terms of x.   [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.